Chem 170

Stoichiometric Calculations

Module Five

Stoichiometric Calculations Using Balanced Chemical Reactions

Introduction to Module Five

In the introduction to Module 4 we presented the following balanced recipe for a cheeseburger.

1 hamburger patty + 1 slice of cheese + 1 English muffin +
3 pickles + 2 slices of onion + 1 squirt of mustard → 1 yummy cheeseburger

This recipe is helpful because it provides precise quantitative instructions for preparing the ideal cheeseburger. For example, you need three pickles and two slices of onion, not three slices of onion and two pickles. The same is true for a chemist interested in synthesizing $K_2[Cu(C_2O_4)_2]•2H_2O$ using copper sulfate, $CuSO_4$, and potassium oxalate, $K_2C_2O_4$. The balanced reaction

$$CuSO_4 + 2K_2C_2O_4 + 2H_2O \rightarrow K_2[Cu(C_2O_4)_2]•2H_2O + K_2SO_4$$

shows that one mole of copper sulfate and two moles of potassium oxalate are needed to prepare one mole of $K_2[Cu(C_2O_4)_2]•2H_2O$.

Sometimes we need to scale-up or scale-down a reaction or recipe. Here, too, a balanced reaction or recipe helps. For example, suppose you are preparing 12 cheeseburgers for a picnic. Because you have the balanced recipe you know that you will need three pickles per cheeseburger, or total of 36 pickles to make 12 cheeseburgers. In the same manner, a chemist who needs to synthesize only 0.010 moles of $K_2[Cu(C_2O_4)_2]•2H_2O$ needs 0.010 mol $CuSO_4$ and 0.020 mol $K_2C_2O_4$. The general applications of stoichiometry are the subject of this module.

Objectives for Module Five

In completing this module you will master the following objectives:

- to determine the moles or grams of one reactant needed to react completely with a known amount of another reactant
- to determine the moles or grams of a reactant needed to prepare a known amount of product
- to determine the expected moles or grams of a product given a known amount of reactant
- to combine stoichiometric calculations with balancing reactions

Stoichiometric Unit Conversion Factors

In Module 1 we discussed a mathematical tool called dimensional analysis. As a reminder, dimensional analysis is an approach to solving problems in which we convert one unit to another by multiplying by a ratio of units that is equivalent to 1. For example, in converting 6 feet to inches, we write

$$6.0\ \cancel{\text{ft}} \times \frac{12\ \text{in}}{1\ \cancel{\text{ft}}} = 72\ \text{in}$$

where the ratio 12 in/1 ft is called a unit conversion factor. In stoichiometric calculations we will make use of two types of unit conversion factors: those showing a stoichiometric ratio from a balanced chemical reaction

$$\frac{2\ \text{mol}\ K_2C_2O_4}{1\ \text{mol}\ K_2[Cu(C_2O_4)_2] \bullet 2H_2O}$$

and those for a compound's molar mass

$$\frac{159.6\ \text{g}\ CuSO_4}{\text{mol}\ CuSO_4}$$

There are three general types of conversions in stoichiometric calculations; these are, in increasing order of complexity

- converting from moles of one compound to moles of another compound, which requires only a stoichiometric ratio
- converting from moles or grams of one compound to grams or moles of another compound, which requires a stoichiometric ratio and one compound's molar mass
- converting from grams of one compound to grams of another compound, which requires a stoichiometric ratio and the molar masses of both compounds

Mole-to-Mole Stoichiometric Calculations

Converting the moles of one species into the moles of another species is the simplest possible calculation, requiring only a single unit conversion factor. Because the stoichiometric coefficients in a balanced chemical reaction inform us of the mole-to-mole ratio for any two species, we can write the appropriate unit conversion factor by inspection. For example, in the introduction we note that it takes 0.020 mol of $K_2C_2O_4$ to make 0.010 mol of $K_2[Cu(C_2O_4)_2]{\bullet}2H_2O$. It is easy to set up a calculation to show this

$$0.010 \text{ mol } K_2[Cu(C_2O_4)_2] \bullet 2H_2O \times \frac{2 \text{ mol } K_2C_2O_4}{1 \text{ mol } K_2[Cu(C_2O_4)_2] \bullet 2H_2O} = 0.020 \text{ mol } K_2C_2O_4$$

Example 1. One of the key reactions in the production of photochemical smog is the oxidation of nitric oxide, NO, to form nitrogen dioxide, NO_2.

$$2NO + O_2 \rightarrow 2NO_2$$

How many moles of NO_2 form when 0.158 mol of O_2 are consumed?

Solution. The only unit conversion factor we need is the stoichiometric relationship between moles of NO and moles of NO_2; thus

$$0.158 \text{ mol } O_2 \times \frac{2 \text{ mol NO}}{1 \text{ mol } O_2} = 0.316 \text{ mol NO}$$

Mole/Gram-to-Gram/Mole Stoichiometric Calculations

Suppose you wish to synthesize 0.010 mol of $K_2[Cu(C_2O_4)_2]{\bullet}2H_2O$. Using the balanced reaction, it is easy to show that this requires 0.010 mol $CuSO_4$ and 0.020 mol $K_2C_2O_4$. A laboratory balance, however, measures amounts in grams, not moles. Calculating the required mass of $CuSO_4$ given the desired moles of $K_2[Cu(C_2O_4)_2]{\bullet}2H_2O$ is easily accomplished using the compound's molar mass; thus

$$0.010 \text{ mol } CuSO_4 \times \frac{159.6 \text{ g } CuSO_4}{\text{mol } CuSO_4} = 1.6 \text{ g } CuSO_4$$

Normally, we combine these two steps – converting moles of $K_2[Cu(C_2O_4)_2]{\bullet}2H_2O$ to moles of $CuSO_4$, and moles of $CuSO_4$ to grams of $CuSO_4$ – into one calculation using two unit conversion factors

$$0.010 \text{ mol } K_2[Cu(C_2O_4)_2] \bullet 2H_2O \times \frac{1 \text{ mol } CuSO_4}{1 \text{ mol } K_2[Cu(C_2O_4)_2] \bullet 2H_2O} \times \frac{159.6 \text{ g } CuSO_4}{\text{mol } CuSO_4} = 1.6 \text{ g } CuSO_4$$

Combining two or more conversions between units in a single stoichiometric calculation is an important skill. These calculations can be done step-by-step, but it is more time consuming. Both approaches are shown in the following example; the module's remaining examples, however, emphasize using a single calculation.

Example 2. Astronauts used to carry canisters of LiOH to remove CO_2 from the closed environment of a space capsule. The balanced reaction for this process is

$$2LiOH + CO_2 \rightarrow Li_2CO_3 + H_2O$$

How many grams of CO_2 are removed by a canister containing 53.3 mol LiOH?

Solution. Two unit conversion factors are need to complete this calculation – the stoichiometry between LiOH and CO_2, and the molar mass for CO_2. Taking the calculation step-by-step, we first calculate the moles of CO_2 equivalent to 53.3 mol LiOH and then convert the moles of CO_2 to grams of CO_2.

$$53.3 \text{ mol LiOH} \times \frac{1 \text{ mol } CO_2}{2 \text{ mol LiOH}} = 26.65 \text{ mol } CO_2$$

$$26.65 \text{ mol } CO_2 \times \frac{44.01 \text{ g } CO_2}{\text{mol } CO_2} = 1.17 \times 10^3 \text{ g } CO_2$$

Of course, we can combine the two steps into a single calculation, as shown here.

$$53.3 \text{ mol LiOH} \times \frac{1 \text{ mol } CO_2}{2 \text{ mol LiOH}} \times \frac{44.01 \text{ g } CO_2}{\text{mol } CO_2} = 1.17 \times 10^3 \text{ g } CO_2$$

The next example shows how a stoichiometric calculation involving a gram-to-mole conversion can be used to compare two reactions.

Example 3. Baking soda, $NaHCO_3$, and milk of magnesia, $Mg(OH)_2$, are two compounds used as antacids to neutralize stomach acid, H^+. The relevant reactions are

$$NaHCO_3 + H^+ \rightarrow Na^+ + H_2O + CO_2 \qquad Mg(OH)_2 + 2H^+ \rightarrow 2H_2O + Mg^{2+}$$

Which antacid neutralizes the most H^+ per gram?

Solution. Using 1.00 g of each antacid, we find that $Mg(OH)_2$ is the most effective.

$$1.00 \text{ g } NaHCO_3 \times \frac{1 \text{ mol } NaHCO_3}{84.01 \text{ g } NaHCO_3} \times \frac{1 \text{ mol } H^+}{1 \text{ mol } NaHCO_3} = 0.0119 \text{ mol } H^+$$

$$1.00 \text{ g } Mg(OH)_2 \times \frac{1 \text{ mol } Mg(OH)_2}{58.33 \text{ g } Mg(OH)_2} \times \frac{2 \text{ mol } H^+}{1 \text{ mol } Mg(OH)_2} = 0.0343 \text{ mol } H^+$$

Gram-to-Gram Stoichiometric Calculations

The most common stoichiometric calculations are those in which the mass of one species is converted to the mass of another species. For example, we many need to calculate the grams of $CuSO_4$ needed to react completely with 0.231 g $K_2C_2O_4$, or the grams of $K_2[Cu(C_2O_4)_2]$•$2H_2O$ expected when completely reacting 0.231 g $K_2C_2O_4$. Such calculations require three unit conversion factors – the molar mass of the first compound, the stoichiometric ratio between the compounds, and the molar mass of the second compound; thus

$$0.231 \text{ g } K_2C_2O_4 \times \frac{1 \text{ mol } K_2C_2O_4}{166.2 \text{ g } K_2C_2O_4} \times \frac{1 \text{ mol } CuSO_4}{2 \text{ mol } K_2C_2O_4} \times \frac{159.6 \text{ g } CuSO_4}{1 \text{ mol } CuSO_4} = 0.111 \text{ g } CuSO_4$$

Example 4. A typical problem in the iron industry is determining how much carbon is needed to reduce the iron in Fe_2O_3 to Fe. The reaction of interest is

$$Fe_2O_3 + 3C \rightarrow 2Fe + 3CO$$

How many grams of C are needed to completely reduce 5.00×10^5 g Fe_2O_3? How many grams of Fe are expected to be produced?

Solution. Working the problem step-by-step, we first calculate the moles of Fe_2O_3 in 5.00×10^5 g

$$5.00 \times 10^5 \text{ g } Fe_2O_3 \times \frac{1 \text{ mol } Fe_2O_3}{159.7 \text{ g } Fe_2O_3} = 3.131 \times 10^3 \text{ mol } Fe_2O_3$$

Next, we calculate the moles of C needed to react with 3.131×10^3 mol Fe_2O_3

$$3.131 \times 10^3 \text{ mol } Fe_2O_3 \times \frac{3 \text{ mol C}}{1 \text{ molFe}_2O_3} = 9.393 \times 10^3 \text{ mol C}$$

Finally, we calculate the grams of C in 9.393×10^3 mol C

$$9.393 \times 10^3 \text{ mol C} \times \frac{12.01 \text{ g C}}{1 \text{ mol C}} = 1.13 \times 10^5 \text{ g C}$$

To find the expected grams of Fe produced from 5.00×10^5 g Fe_2O_3 we will combine the three steps into a single calculation; thus

$$5.00 \times 10^5 \text{ g } Fe_2O_3 \times \frac{1 \text{ mol } Fe_2O_3}{159.7 \text{ g } Fe_2O_3} \times \frac{2 \text{ mol Fe}}{1 \text{ mol } Fe_2O_3} \times \frac{55.85 \text{ g Fe}}{1 \text{ mol Fe}} = 3.50 \times 10^5 \text{ g Fe}$$

As shown in the next example, it is possible to carry out stoichiometric calculations over two (or more) reactions, provided that a product of one reaction is a reactant in the following reaction.

Example 5. In Example 2 we saw how LiOH is used to remove CO_2 from the atmosphere of a closed space capsule. Another approach is use potassium superoxide, KO_2. As shown by the following two reactions

$$4KO_2 + 2H_2O \rightarrow 3O_2 + 4KOH$$

$$KOH + CO_2 \rightarrow KHCO_3$$

using KO_2 allows for the removal of both CO_2 and H_2O, while also providing additional O_2 for breathing. When spent, the canister, which originally contained only KOH, contains only $KHCO_3$. Suppose that a typical canister contains 500.0 g KO_2. How much will the canister's contents weigh after absorbing as much H_2O and CO_2 as it can?

Solution. We begin by calculating the grams of KOH in the canister when the all the KO_2 has reacted.

$$500.0 \text{ g KO}_2 \times \frac{1 \text{ mol KO}_2}{71.10 \text{ g KO}_2} \times \frac{4 \text{ mol KOH}}{4 \text{ mol KO}_2} \times \frac{56.11 \text{ g KOH}}{1 \text{ mol KOH}} = 394.59 \text{ g KOH}$$

Next, we convert the mass of KOH to the mass of $KHCO_3$.

$$394.59 \text{ g KOH} \times \frac{1 \text{ mol KOH}}{56.11 \text{ g KOH}} \times \frac{1 \text{ mol KHCO}_3}{1 \text{ mol KOH}} \times \frac{100.1 \text{ g KHCO}_3}{1 \text{ mol KHCO}_3} = 703.9 \text{ g KHCO}_3$$

Note that these two calculations can be combined into a single calculation by using all four-unit conversion factors.

$$500.0 \text{ g KO}_2 \times \frac{1 \text{ mol KO}_2}{71.10 \text{ g KO}_2} \times \frac{4 \text{ mol KOH}}{4 \text{ mol KO}_2} \times \frac{1 \text{ mol KHCO}_3}{1 \text{ mol KOH}} \times \frac{100.1 \text{ g KHCO}_3}{1 \text{ mol KHCO}_3} = 703.9 \text{ g KHCO}_3$$

Combining Balancing Reactions and Stoichiometric Calculations

The last topic for this module combines the lessons learned here with the lessons of Module 4. Be sure to verify that the reactions provided to you are balanced, or write a balanced reaction if you are provided only with a verbal description.

Example 6. When octane, C_8H_{18}, is burned in the presence of excess O_2, the usual products of carbon dioxide, CO_2, and water, H_2O, form. Write a balanced chemical equation for this reaction and calculate the grams of H_2O produced from the combustion of 7.90×10^2 g C_8H_{18}.

Solution. The unbalanced reaction is

$$C_8H_{18} + O_2 \rightarrow CO_2 + H_2O$$

Balancing C and H gives

$$C_8H_{18} + O_2 \rightarrow 8CO_2 + 9H_2O$$

Next, balancing O gives

$$2C_8H_{18} + 25O_2 \rightarrow 16CO_2 + 18H_2O$$

Finally, we convert the mass of C_8H_{18} to the mass of H_2O

$$7.90\times 10^2 \text{ g } C_8H_{18} \times \frac{1 \text{ mol } C_8H_{18}}{114.2 \text{ g } C_8H_{18}} \times \frac{18 \text{ mol } H_2O}{2 \text{ mol } C_8H_{18}} \times \frac{18.02 \text{ g } H_2O}{1 \text{ mol } H_2O} = 1.12\times 10^3 \text{ g } H_2O$$

Practice Problems

The following problems provide practice in meeting this module's objectives. Answers are provided on the last page. Be sure to seek assistance if you experience difficulty with any of these problems. When you are ready, schedule an appointment for the module exam.

1. During his or her lifetime, the average American uses approximately 795 kg of Cu in the form of coins, plumbing pipes, and electrical wiring. Most of this copper is obtained from sulfide ores, such as chalcocite, Cu_2S. To obtain the copper metal, the chalcocite is first roasted (heated in the presence of oxygen), forming a copper oxide, Cu_2O. The balanced reaction for this process is

$$2Cu_2S + 3O_2 \rightarrow 2Cu_2O + 2SO_2$$

   How many moles of O_2 are needed to roast 10.0 mol of Cu_2S? How many grams of SO_2 are formed when roasting 10.0 mol of Cu_2S? How many kilograms of O_2 are required to form 2.68 kg Cu_2S?

2. Hydrofluoric acid, HF, cannot be stored in glass containers because it reacts with the silicates in glass. The relevant reaction is

$$Na_2SiO_3 + 8HF \rightarrow H_2SiF_6 + 2NaF + 3H_2O$$

   How many moles of HF are required to dissolve 0.500 mol Na_2SiO_3? How many grams of Na_2SiO_3 can be dissolved by 0.500 mol HF?

3. Aspirin, $C_9H_8O_4$, is synthesized by the following reaction between salicylic acid, $C_7H_6O_3$, and acetic anhydride, $C_4H_6O_3$

$$C_7H_6O_3 + C_4H_6O_3 \rightarrow C_9H_8O_4 + C_2H_4O_2$$

   The other product of the reaction is acetic acid. How many grams of acetic anhydride are needed to completely react with 1.00×10^2 g of salicylic acid? How many grams of aspirin are expected as a product?

4. In 1774, the British chemist Joseph Priestly prepared oxygen, O_2, for the first time by heating a sample of mercury oxide, HgO.

$$2HgO \rightarrow 2Hg + O_2$$

   Mercury is another product of the reaction. Suppose a sample of HgO completely decomposes, producing 6.47 g of O_2. How many grams of Hg are obtained?

5. In a catalytic converter, the gases NO (a major pollutant) and CO (very toxic) are converted to the harmless gases N_2 and CO_2. The reaction is

$$2NO + 2CO \rightarrow N_2 + 2CO_2$$

How much NO gas is needed to completely react with 100.0 g CO?

6. Industrial plants burning "dirty" coal as a source of energy generate foul smelling sulfur dioxide, SO_2, as a by-product. The reaction responsible for this is

$$S + O_2 \rightarrow SO_2$$

If a plant burns 3.00×10^2 kg of coal that is 0.5% S by mass, how many kg of sulfur dioxide are produced?

7. The surface atoms of aluminum metal corrode in air to form an impervious aluminum oxide coating that prevents further corrosion. The oxidation reaction is

$$4Al + 3O_2 \rightarrow 2Al_2O_3$$

How many micrograms of Al_2O_3 form when 10.0 μg of Al undergo oxidation? How many atoms of Al is this?

8. Camels store the fat tristearin, $C_{57}H_{110}O_6$, in their humps. In addition to being a source of energy, the fat also serves as a source of water because the fat's oxidation

$$2C_{57}H_{110}O_6 + 163O_2 \rightarrow 114CO_2 + 110H_2O$$

generates H_2O as a product. How many kilograms of water can a camel obtain from 2.5 kg of tristearin?

9. Thermite is a mixture of Fe_2O_3 and Al powder that was once used to weld railroad tracks. A portion of the powdered mixture is placed on the pieces to be welded together and ignited using a fuse. The resulting reaction, which is shown here

$$Fe_2O_3 + 2Al \rightarrow Al_2O_3 + 2Fe$$

is spectacular, producing molten iron and aluminum oxide as products. How many grams of Fe form when 13.5 g Al react completely?

10. An intermediate step in the industrial production of nitric acid, HNO_3, is the reaction of ammonia, NH_3, with oxygen gas, O_2, to form nitrogen monoxide, NO, and water,

H_2O. Write a balanced chemical equation for this reaction and determine how many grams of NO can form by the reaction of 466 g NH_3.

11. Problem 1 shows the first step in extracting Cu from Cu_2S

$$2Cu_2S + 3O_2 \rightarrow 2Cu_2O + 2SO_2$$

In the second step, the Cu_2O reacts with C, producing copper metal and carbon monoxide

$$Cu_2O + C \rightarrow 2Cu + CO$$

How many grams of Cu can be produced from every 1.0 grams of Cu_2S?

12. Upon heating, carbonate and bicarbonate minerals decompose to produce oxides and carbon dioxide. For example, here are the balanced reactions for the decomposition of three minerals

$$2KHCO_3 \rightarrow K_2O + H_2O + 2CO_2$$

$$Na_2CO_3 \rightarrow Na_2O + CO_2$$

$$Mn(CO_3)_2 \rightarrow MnO_2 + 2CO_2$$

Which of these three minerals produces the most CO_2 per gram of mineral and, to two significant figures, how many grams of CO_2 does it produce per gram of mineral?

13. One source of the element molybdenum is an ore containing molybdenum sulfide, MoS_2. The ore is “roasted” by heating in the presence of oxygen, producing molybdenum oxide, MoO_3, and sulfur dioxide, SO_2.

$$MoS_2 + O_2 \rightarrow MoO_3 + SO_2$$

Upon its emission into the environment, sulfur dioxide reacts with other atmospheric gases to produce sulfuric acid, H_2SO_4, one source of acid rain.

$$SO_2 + O_2 + H_2O \rightarrow H_2SO_4$$

Balance both reactions and determine how much sulfuric acid is expected from the roasting of 5.00×10^2 kg of MoS_2.

14. Smoke screens used by military are produced by mixing titanium tetrachloride, $TiCl_4$, with water, H_2O, producing titanium dioxide, TiO_2, and hydrogen chloride, HCl, as

products. Write a balanced chemical equation for this reaction and determine how many grams of titanium dioxide can be made from 10.0 g of titanium tetrachloride.

Answers to Practice Problems

1. 15.0 mol O_2, 641 g SO_2, 0.808 kg O_2

2. 4.00 mol HF, 7.63 g Na_2SiO_3

3. 73.9 g $C_4H_6O_3$, 1.30×10^2 g $C_9H_8O_4$

4. 81.1 g Hg

5. 107.1 g NO

6. 3.00 kg SO_2

7. 18.9 μg Al_2O_3, 2.23×10^{17} atoms of Al

8. 2.8 kg H_2O

9. 27.9 g Fe

10. $4NH_3 + 5O_2 \rightarrow 4NO + 6H_2O$, 821 g NO

11. 0.80 g Cu

12. $Mn(CO_3)_2$, 0.50 g CO_2/g mineral

13. $2MoS_2 + 7O_2 \rightarrow 2MoO_3 + 4SO_2$, $2SO_2 + O_2 + 2H_2O \rightarrow 2H_2SO_4$, 613 kg H_2SO_4

14. $TiCl_4 + 2H_2O \rightarrow TiO_2 + 4HCl$, 4.21 g TiO_2